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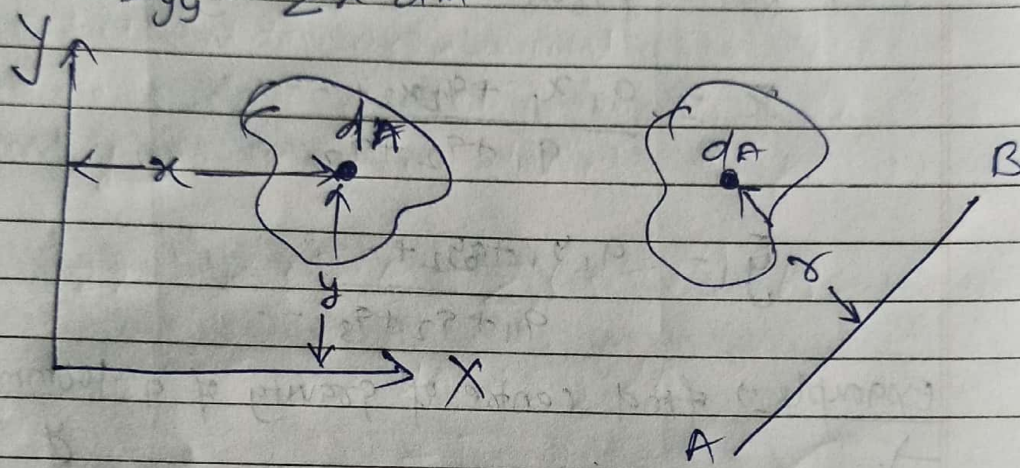
Engineering Mechanics (16/02/04)

Moment of Inertia:-

Consider the area 'dA' is an elemented area with co-ordinates as x and y. The term $\Sigma y^2 dA$ is called moment of inertia of the area about x axis and denoted as I_{xx}.

Similarly moment of Inertia about y axis is $I_{yy} = \Sigma x^2 dA$.



In general if r is the distance of elemented area dA from the axis AB (2nd fig), the sum of terms $\Sigma r^2 dA$ cover the entire area is called moment of inertia of the area about axis AB

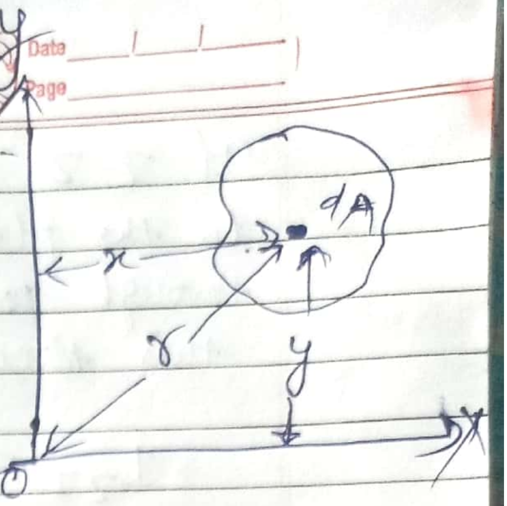
$$\therefore I_{AB} = \Sigma r^2 dA = \int r^2 dA$$

The term r.dA is called moment of Area. Therefore the term $r^2 dA$ may be called as moment of moment of Area or second moment of Area.

* polar moment of Inertia (J or I_{zz}):—

The moment of Inertia about an axis perpendicular to the plane of an area is known as polar moment of Inertia. It may be denoted as J or I_{zz}.

Thus the moment of Inertia about an axis perpendicular to the plane of Area about 'O' is called polar moment of Inertia at 'O' and given by



$$\boxed{I_{zz} = \Sigma r^2 dA}$$

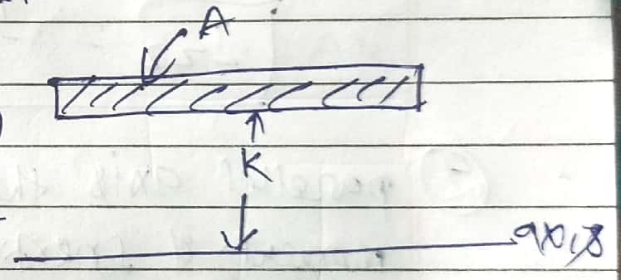
* Radius of Gyration :-
It is a methematical term and defined by relation

$K = \sqrt{I/A}$

K = radius of gyration
I = moment of Inertia
A = cross-sectional Area.

$\therefore K_{xx} = \sqrt{\frac{I_{xx}}{A}}$

$K_{yy} = \sqrt{\frac{I_{yy}}{A}}$ & $K_{AB} = \sqrt{\frac{I_{AB}}{A}}$



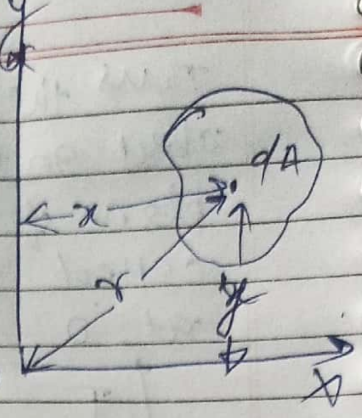
from fig, $\boxed{I = AK^2}$

There are two theorems of moment of Inertia :

1. perpendicular axis theorem
2. parallel axis theorem :-

The moment of Inertia of an area about an axis perpendicular to its plane (polar moment of Inertia) at any point 'O' is equal to the sum of moments of Inertia about any two mutually perpendicular axis through the same point 'O' and lying in the plane of the area.

If Z-Z is the axis normal to the plane of paper passing through point 'O' then A/C to this theorem,



$$I_{ZZ} = I_{xx} + I_{yy} \quad \text{— (1)}$$

proof:

Let us consider an elemental area 'dA' at a distance r from 'O'. Let co-ordinates of dA be x and y then from definition of I^r axis

$$I_{ZZ} = \sum r^2 dA = \sum (x^2 + y^2) dA$$

$$I_{ZZ} = \sum x^2 dA + \sum y^2 dA$$

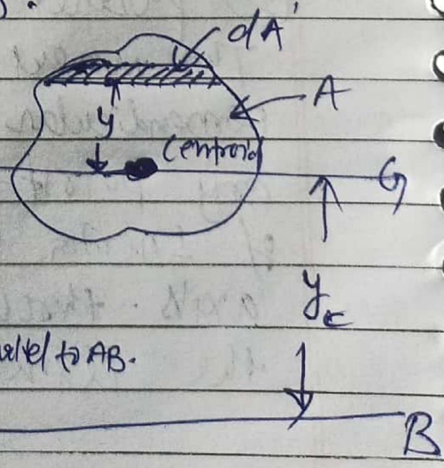
$$\boxed{I_{ZZ} = I_{xx} + I_{yy}}$$ proved

(2) parallel axis theorem:-

Moment of inertia about any axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between the two parallel axes.

∴ from this theorem,

$$\boxed{I_{AB} = I_{GG} + A y_c^2} \quad \text{— (1)}$$



I_{AB} = M.I about an axis AB.

I_{GG} = M.I about centroidal axis GG parallel to AB.

A = Area of plane fig.

y_c = distance b/w axis AB & centroidal axis GG.

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proof :-

consider an elemental ~~area dA~~ parallel strip dA at a distance y from centroidal axis

then,

$$I_{AB} = \sum (y+y_c)^2 dA = \sum (y^2 + 2yy_c + y_c^2) dA$$

$$= \sum y^2 dA + \sum 2yy_c\, dA + \sum y_c^2 dA$$

Now,

$\sum y^2 dA$ = moment of Inertia about the axis $GG = \underline{I_{GG}}$

$$\sum 2yy_c\, dA = 2y_c \sum y\, dA = 2y_c A \frac{\sum y\, dA}{A}$$

the term $2y_c A$ is constant and $\frac{\sum y\, dA}{A}$ is the distance of centroid from reference axis GG.

Since GG is passing through the centroid itself is zero hence the term $\sum 2yy_c dA$ is zero.

& third term, $\sum y_c^2 dA = y_c^2 \sum dA = A y_c^2$

$$\therefore \boxed{I_{AB} = I_{GG} + A y_c^2}$$ proved

Formula

(1) M.I of Rectangle about centroidal axis $= \frac{bd^3}{12}$

(2) M.I of triangle about base $= \frac{bh^3}{12}$

(3) M.I of circle about its diametral axis $= \frac{\pi d^4}{64}$

(4) " Hollow circle " " $= \frac{\pi}{64}(D_1^4 - D_2^4)$